Let $f$ be a candidate function, and let $g$ be a reference function. The definition of $f = o(g)$ is $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$.

1. Write corresponding definitions for
   
   (a) $f = O(g)$
   (b) $f = \Theta(g)$
   (c) $f = \Omega(g)$
   (d) $f = \omega(g)$

2. Prove that $\log n = O(n)$.
3. Prove that $n = O(n \log n)$.
4. Prove that $n \log n = O(n^2)$.
5. Prove that $2^n = \Omega(5^{\log n})$.
6. Prove that $\log^3 n = o(n^{1/2})$.

Find the asymptotic complexity class of $T(n)$ for each of the following recurrences. That is, complete the expression $T(n) = \Theta(?)$. In each case, prove your claim using the master template or other recurrence techniques in case the master template is not applicable. The notation $\log n$ refers to $\log_2 n$. Assume that expressions that should evaluate to integers contain an implied round-down function. For example, $T(n/3)$ means $T(\lfloor n/3 \rfloor)$ when necessary.

7. $T(n) = 2T(n/3) + 1$
8. $T(n) = 5T(n/4) + n$
9. $T(n) = 7T(n/7) + n$
10. $T(n) = 9T(n/3) + n^2$
11. $T(n) = 8T(n/2) + n^3$
12. $T(n) = 49T(n/25) + n^{3/2} \log n$
13. $T(n) = T(n - 1) + 2$
14. $T(n) = 2T(n - 1) + 1$
15. (optional)
    
    $T(n) = T(n^{1/2}) + 1$