1. Prove that $\log_2(n!) = \Theta(n \log_2 n)$.

2. Prove that $n! = o(n^n)$.

3. Prove that $n! = \omega(2^n)$.

4. Suppose $f : \mathbb{Z}^+ \rightarrow \mathbb{R}^+$ and also
   
   - $\exists K_1 > 0$ such that $f(n) \leq K_1 n$, for all but finitely many $n$,
   - $\exists K_2 > 0$ such that $f(n) \geq K_2 \sqrt{n}$, for infinitely many $n$.

   (a) Prove that $f(n) = o(n^2)$.

   (b) Give a counterexample to show that $f(n) = \omega(\sqrt{n})$ is not necessarily true.

5. Consider the reference function $g(n) = n^3 \lg n$ and the related asymptotic classes of the following figure. For each of the five areas, A through E, give a function that bears the appropriate relationship to $g(n)$. Note that area B contains those functions $f$ such that $f \in O(g)$, but $f \notin o(g)$ and $f \notin \Theta(g)$. Similarly, area D contains that functions $f \in \Omega(g) \setminus (\omega(g) \cup \Theta(g))$.

Each of the exercises below involves a choice among the master theorem templates discussed in lecture. For each, indicate which case applies and specify the asymptotic growth class of the function. If no case applies, solve the recurrence in some other manner.

6. $T(n) = 2T(\lfloor n/4 \rfloor) + n^{1/2}$.

7. $T(n) = 3T(\lfloor n/2 \rfloor) + n \lg n$.

8. (optional) $T(n) = 5T(\lfloor n/5 \rfloor) + \frac{n}{\lg n}$. This one is difficult, but you should be able to show that the master theorem is not applicable.

9. $T(n) = 4T(\lfloor n/2 \rfloor) + n^2 \sqrt{n}$.

10. (optional) $T(n) = 2T(\lfloor n/2 \rfloor) + n \lg n$. This one is difficult, but you should be able to show that the master theorem is not applicable.